CMSI 282 Problem Set #1
Due January 29, 2009

1) Prove:

   a) $\log_b a = 1 / (\log_a b)$

   b) $\log_c a = (\log_c b) (\log_b a)$

   c) $\log_b(xy) = (\log_b x) + (\log_b y)$

   d) $a^{\log b} = b^{\log a}$

   e) for any integer $n \geq 0$, $1^2 + 2^2 + ... + n^2 = n(n+1)(2n+1) / 6$

2) Which of these recurrences can be classified using the Master Theorem?

   a) $T_a(n)$ = *if* n = 1 *then* 1 *else* $2T_a(n-1) + (n-2)$

   b) $T_b(n)$ = *if* n = 1 *then* 1 *else* $2T_b(n/3) + (n-2)$

   c) $T_c(n)$ = *if* n = 1 *then* 1 *else* $3T_c(n/2) + 5n^2$

   d) $T_d(n)$ = *if* n = 1 *then* 1 *else* $2T_d(n-1) + 3T_d(n-2)$

   e) $T_e(n)$ = *if* n = 1 *then* 5 *else* $16T_e(n/3) + 5n^3$

3) Classify these recurrences using big-theta notation. In each case, indicate how you arrived at your conclusion (e.g., MASTER THEOREM, or, REPEATED SUBSTITUTION, etc.):

   a) $T_a(n)$ = *if* n = 1 *then* 1 *else* $T_a(n-1) + (n-1)$

   b) $T_b(n)$ = *if* n = 1 *then* 1 *else* $2T_b(n/3) + (n-2)$

   c) $T_c(n)$ = *if* n = 1 *then* 1 *else* $3T_c(n/2) + 5n^2$

   d) $T_d(n)$ = *if* n = 1 *then* 1 *else* $16T_d(n/3) + 5n^2$

   e) $T_e(n)$ = *if* n = 1 *then* 1 *else* $16T_e(n/3) + 5n^3$

   f) $T_f(n)$ = *if* n = 1 *then* 1 *else* $16T_f(n/4) + 5n^4$

   g) $T_g(n)$ = *if* n = 1 *then* 1 *else* $3T_g(n/2) + 5$

h) $T_h(n)$ = *if* n = 1 *then* 1 *else* $16T_h(n/2) + 5n^4$

i) T_i (n) = *if* n = 1 *then* 1 *else* $T_i(n/3) + 2$

j) $T_j(n)$ = *if* n = 1 *then* 1 *else* $3T_j(n/2)$

k) $T_k(n)$ = *if* n = 1 *then* 1 *else* $T_k(n/2)$

l) $T_l(n)$ = *if* n = 1 *then* 10 *else* $nT_l(n/2)$

m) $T_m(n)$ = *if* n = 1 *then* 10 *else* $16T_m(n/16)$

n) T(n) = *if* n = 1 *then* 1 *else* 2T(n-1) + 1

o) $T_o(n)$ = *if* n = 1 *then* 1 *else* $T_o(n-1) + n^2$ (hint: see problem 1d, above)

p) $T_p(n)$ = *if* n = 1 *then* 1 *else* $2T_p(n/2) + n^{1.5}$

q) $T_q(n)$ = *if* n = 1 *then* 10 *else* $16T_q(n/16) + n$

r) $T_r(n)$ = *if* n = 1 *then* 10 *else* $16T_r(n/17) + n$